



# The decay constants $f_{B^+}$ and $f_{D^+}$ from three-flavor lattice QCD

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## Fermilab Lattice, MILC and HPQCD Collaborations

We present new results for  $f_{B^+}$  and  $f_{D^+}$  from the MILC 2 + 1 flavor  $a = 0.09$  fm “fine” lattice. We use clover heavy quarks in the Fermilab interpretation and improved staggered light quarks. Lattice results from partially quenched QCD fix the parameters of staggered chiral perturbation theory which is used in the extrapolation to the physical decay constants.

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$a$ [fm]	$am_h$	$am_l$	$\beta$	$r_1/a$	configs	# $m_q$
0.09	0.031	0.0031	7.08	3.71(1)	435	11
		0.0062	7.09	3.69(1)	557	8
		0.0124	7.11	3.71(1)	518	10

**Table 1:** Properties of the MILC fine lattice gauge ensembles and the number of light valence quarks.

## 1. Introduction

When compared to precise experimental results, the  $D$  meson decay constants are a critical check of lattice methods need for  $f_B$ . In Reference [1] we predicted  $f_{D^+} = 201 \pm 3 \pm 17$  MeV in good agreement with the CLEO measurement  $f_{D^+} = 223 \pm 17 \pm 3$  MeV revealed just days later [2]. Our value  $f_{D_s} = 249 \pm 3 \pm 16$  MeV also agrees with the recent experimental result  $282 \pm 16 \pm 7$  from CLEO [3].

The analysis in Reference [1] is based on results from the MILC  $a = 0.12$  fm “coarse” lattice and a subset of present results from the  $a = 0.09$  fm “fine” lattice. Since then, we have significantly extended the analysis of the fine lattice: we have computed  $f_B$  on all three ensembles listed in Table 1, extended the  $D$  meson calculation to the  $\beta = 7.08$  ensemble, and increased the number of light valence quark mass values from five to ten for the  $\beta = 7.11$  ensemble.

Numerical work continues for the coarse lattice. Statistics are being increased at two lightest sea quark masses, the  $D$  analysis is being extended to a second quark mass near charm, and  $B$  meson computations are underway. Decay constant computations are also being repeated at a third, coarser lattice spacing,  $a = 0.15$  fm, to investigate finite lattice spacing effects.

We are presently computing nonperturbative renormalization factors for the light-quark vector current at all three lattice spacings. We refrain from quoting values for the decay constants until the renormalizations are complete. This work instead concentrates on ratios of decay constants from the fine lattice. We will present a complete analysis including all three lattice spacings in future publications.

New in this work are ratios of  $B$  meson decay constants. Precise determinations of  $f_B$ ,  $f_{B_s}$  and the ratio  $f_{B_s}/f_B$  are inputs used to study the Standard Model picture of  $B$ - $\bar{B}$  and  $B_s$ - $\bar{B}_s$  mixing. A study of the mixing matrix elements on the MILC lattices is being presented at this conference [4].

## 2. Simulation details

Properties of the MILC  $a = 0.09$  fm three-flavor gauge ensembles used in this analysis are listed in Table 1. The mass of the two equal-mass light sea quarks is  $m_l$ . The third, heavier sea quark has a mass  $m_h$  around the strange quark mass. Upsilon spectroscopy tells us the heavy quark potential scale  $r_1 = 0.317(7)$  fm [5].

Staggered valence quarks have masses in the range  $0.1m_h \leq m_q \leq m_h$ . The last column of Table 1 lists the number of valence quark mass values. For each of the charm and bottom systems, a single chiral extrapolation (see Sect. 3) is done taking as inputs the heavy-light decay constants computed at all 29 combinations of valence and sea quarks.

The decay constant  $f_{H_q}$  for a meson  $H_q$  is defined by

$$\langle 0 | A_\mu | H_q(p) \rangle = i f_{H_q} p_\mu . \quad (2.1)$$

The combination  $\phi_{H_q} = f_{H_q} \sqrt{m_{H_q}}$  emerges from a combined fit to 2-pt functions

$$C_O(t) = \langle O_{H_q}^\dagger(t) O_{H_q}(0) \rangle \quad (2.2)$$

$$C_{A_4}(t) = \langle A_4(t) O_{H_q}(0) \rangle , \quad (2.3)$$

where  $O_{H_q}$  is a smeared or local pseudoscalar meson operator.

The axial current renormalization is taken to be

$$Z_{A_4}^{Qq} = \rho_{A_4}^{Qq} \sqrt{Z_{V_4}^{QQ} Z_{V_4}^{qq}} . \quad (2.4)$$

The factors  $Z_{V_4}^{ff}$  are computed nonperturbatively while the factor  $\rho_{A_4}^{Qq}$  is known to one-loop order and is close to unity [6].

### 3. NLO Staggered $\chi$ PT

With staggered quarks the (squared) pseudoscalar meson masses are split

$$M_{ab,\xi}^2 = (m_a + m_b)\mu + a^2 \Delta_\xi , \quad (3.1)$$

where  $m_a, m_b$  are quark masses, and the (sixteen) mesons are labeled by their taste representation  $\xi = P, A, T, V, I$  and  $\Delta_P = 0$ .

At next to leading order in  $\chi$ PT the expression for the decay constants is

$$\phi_{H_q} = \Phi_H [1 + \Delta f_H(m_q, m_l, m_h) + p_H(m_q, m_l, m_h)] \quad (3.2)$$

where  $m_h$  and  $m_l$  are sea quark masses and  $m_q$  is the valence quark mass.

With staggered quarks, the logarithmic terms are [7]:

$$\Delta f_H = -\frac{1 + 3g_{H^*H\pi}^2}{2(4\pi f_\pi)^2} [\bar{h}_q + h_q^I + a^2 (\delta'_A h_q^A + \delta'_V h_q^V)] . \quad (3.3)$$

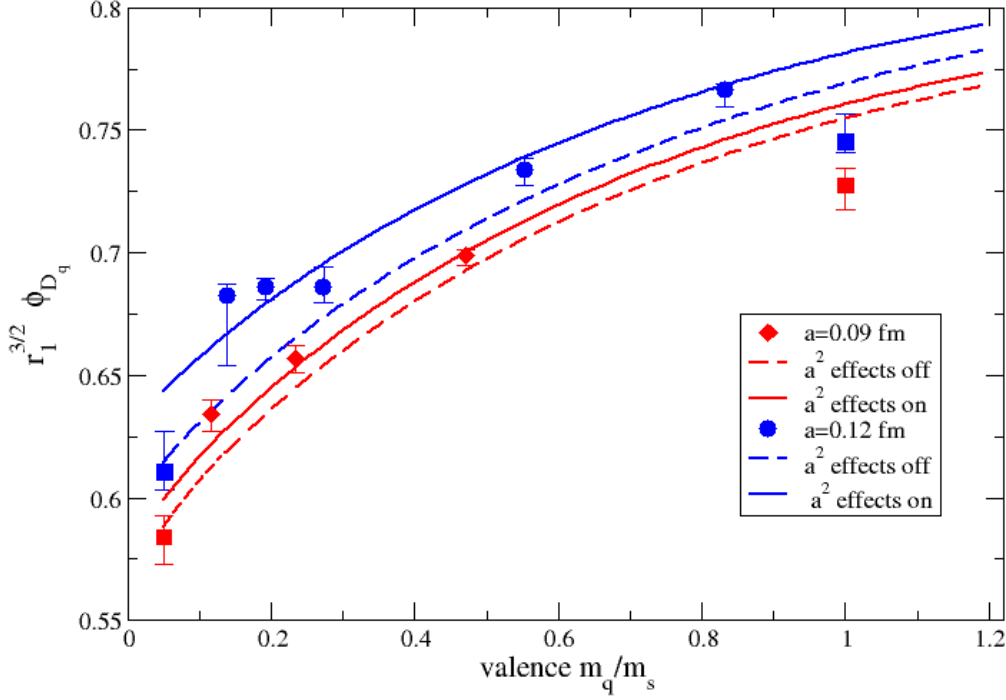
Taste breaking effects arise in the  $\bar{h}_q, h_q^\xi$  functions at finite  $a$  from the meson mass splittings and the  $\delta'_A$  and  $\delta'_V$  terms. Finite  $a$  effects dilute the logarithmic behavior, however, the QCD “chiral logarithm” is recovered in the continuum limit.

The analytic terms are

$$p_H = \frac{1}{2(4\pi f_\pi)^2} [p_1(m_l, m_h) + p_2(m_q)] \quad (3.4)$$

$$p_1 = f_1(\Lambda_\chi) \left[ \frac{11}{9} \mu (2m_l + m_h) + a^2 \left( \frac{3}{2} \bar{\Delta} + \frac{1}{3} \Delta_I \right) \right] \quad (3.5)$$

$$p_2 = f_2(\Lambda_\chi) \left[ \frac{5}{3} \mu m_q + a^2 \left( \frac{3}{2} \bar{\Delta} - \frac{2}{3} \Delta_I \right) \right] , \quad (3.6)$$



**Figure 1:** Chiral extrapolation for  $f_{D^+} f_{D_s}$  at  $a = 0.09$  fm (red) and at  $a = 0.12$  fm (blue). Only points along the  $m_q = m_l$  direction are shown. Error bars are statistical only.

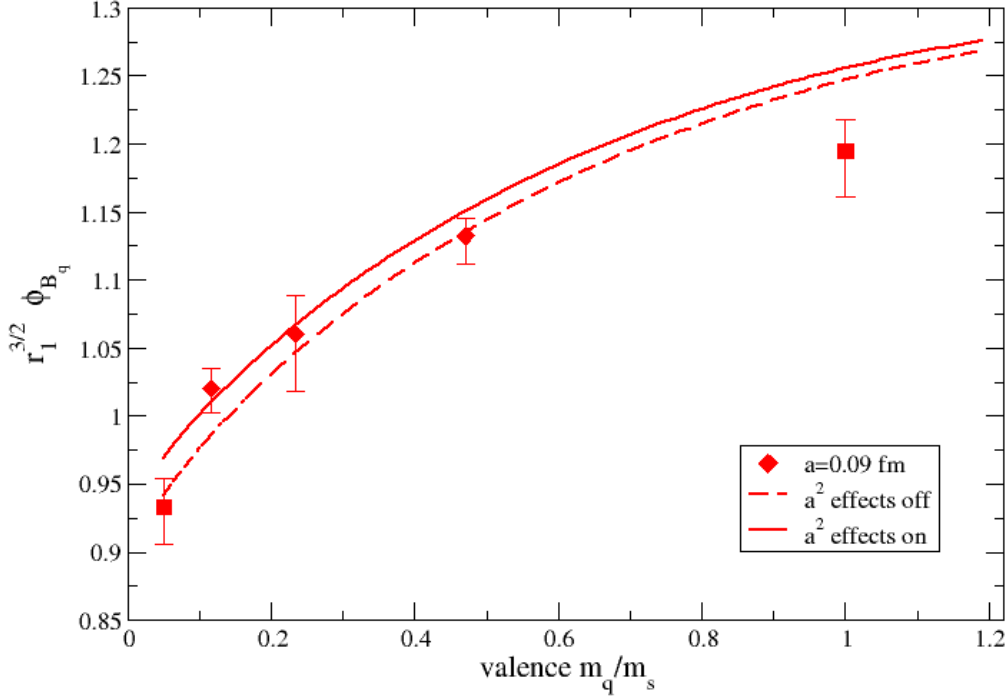
where  $\bar{\Delta} = \frac{1}{16} \sum_{\xi} n_{\xi} \Delta_{\xi}$  and  $n_{\xi} = 1, 4, 6, 4, 1$  for  $\xi = P, A, T, V, I$ . The  $O(a^2)$  terms ensure that the  $\Lambda_{\chi}$  dependence in  $f_1$  and  $f_2$  cancels that of  $\Delta f_q$ .

Equation (3.2) parameterizes our chiral extrapolations. We fit all  $\phi_{H_q}$  at each lattice spacing to determine the parameters. Constraints (value and width) for  $\mu$ ,  $\Delta_{\xi}$ ,  $f_{\pi}$ ,  $\delta'_A$  and  $\delta'_V$  come from  $\chi$ PT for lattice pions and kaons [8]. The coupling  $g_{D^*D\pi}^2$  is constrained by the CLEO measurement. From heavy-quark flavor symmetry we expect  $g_{B^*B\pi}^2 \approx g_{D^*D\pi}^2$ . The remaining parameters  $\Phi$ ,  $f_1$  and  $f_2$  are determined in the fit.

To obtain physical results, we set  $\Delta_{\xi} = \delta'_{A,V} = 0$ ,  $m_h \rightarrow m_s$  and  $m_l \rightarrow (m_u + m_d)/2$ . Then  $\phi_{H_d}$  ( $\phi_{H_s}$ ) is found in the limit  $m_q \rightarrow m_d$  ( $m_s$ ).

#### 4. D meson decay constants

Figure 1 shows separate  $D$  meson chiral extrapolations for the coarse and fine lattices. A total of 60  $\phi_{D_q}$  values corresponding to different combinations of  $(m_q, m_l)$  are fit for the coarse lattice and 29 combinations for the fine lattice. For each extrapolation, only the subset of points along the  $m_q = m_l$  direction is shown. Each solid curve denotes the fit *including*  $a^2$  effects while the broken curve shows the fit after the  $a^2$  effects arising in Equations (3.1, 3.3, 3.6) have been set to zero.



**Figure 2:** Chiral extrapolation for  $f_B$  and  $f_{B_s}$  at  $a = 0.09$  fm. Only statistical errors are shown.

Physical values of the decay constants correspond to the filled squares. The physical  $\phi_{D_s}$  values have been projected into the plane of the figure hence they do not lie on top of the fits in this view.

The extrapolations in Figure 1 are consistent with our published results. We refrain from updating values for  $f_{D^+}$  and  $f_{D_s}$  until we finish numerical work at the three lattice spacings and have a complete analysis of systematic effects.

Many systematic effects, common to  $f_D$  and  $f_{D_s}$ , are expected to cancel in the ratio:

$$f_{D_s}/f_{D^+} = 1.21 \pm 0.01 \pm 0.04 \quad . \quad (4.1)$$

The central value and statistical error are from the fine lattice. The ratio from the coarse lattice agrees within statistical errors. The systematic (second) uncertainty we take to be mainly due to the chiral extrapolation.

## 5. B meson decay constants

In Figure 2 we present a preliminary chiral extrapolation for the  $B$  meson from the fine lattice. Again, fitted points corresponding to unequal  $m_q$  and  $m_l$  are not visible in this view along the  $m_q = m_l$  direction. Points corresponding to the physical decay constants are indicated by filled squares. The point corresponding to  $f_{B_s}$  has been projected into the plane of the figure.

The ratio of decay constants is:

$$f_{B_s}/f_{B^+} = 1.27 \pm 0.02 \pm 0.06 . \quad (5.1)$$

The chiral extrapolation is a dominant source of uncertainty.

We anticipate many common systematic effects will tend to cancel in ratios of  $B$  to  $D$  decay constants. In particular, the common factor  $Z_V^{qq}$  cancels. Statistical correlations among the decay constants are propagated through the error analysis by the bootstrap procedure. We consider the ratios:

$$f_{B^+}/f_{D^+} = 0.95 \pm 0.03 \pm 0.06 \quad (5.2)$$

$$f_{B_s}/f_{D_s} = 0.99 \pm 0.02 \pm 0.06 \quad (5.3)$$

$$R = (f_{B_s}/f_{B^+})/(f_{D_s}/f_{D^+}) = +1.04 \pm 0.01 \pm 0.02 . \quad (5.4)$$

The systematic uncertainties for the first two ratios are dominated by a combination of effects from the uncertainty in tuning of the input bare  $m_c$  and  $m_b$  masses, heavy quark discretization effects and uncertainties arising from the chiral extrapolations. A more detailed analysis of systematic effects is underway. Many systematic effects are further reduced in the double ratio Eq. (5.4).

The deviation of the double ratio in Eq. (5.4) from unity is a measure of both  $SU(3)$  and heavy-quark flavor symmetry breaking among the decay constants, so it is expected to be small:  $R - 1 = O(m_s/m_b - m_s/m_c)$ . Reference [9] estimated the chiral log contribution to be around  $-3.3\%$ . Equation (5.4) indicates the analytic terms are about twice as large and positive in sign.

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